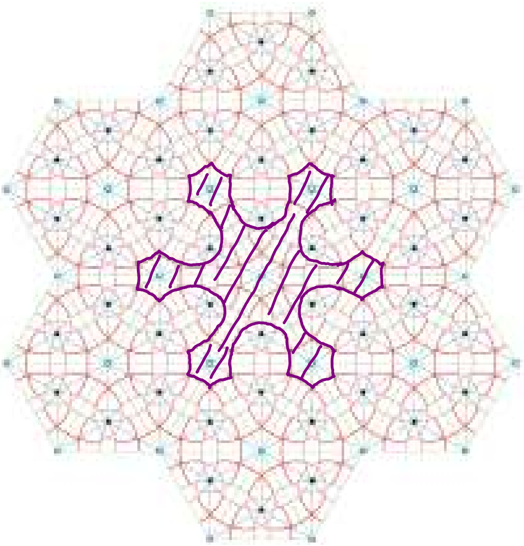
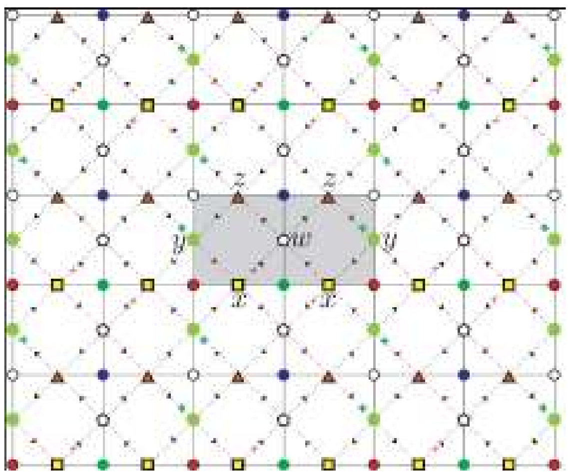
Mirror construction using non-commutative geometry

Joint work with Cheol-Hyun Cho and Hansol Hong





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I. Brief review on mirror construction.

II. Non-commutative mirror construction.

III. Concrete geometries

- **i. Sklyanin algebra and its central element.**
- **ii. Mirror of the pillowcase.**
- **iii. CY threefolds associated to SL(2) Hitchin system.**

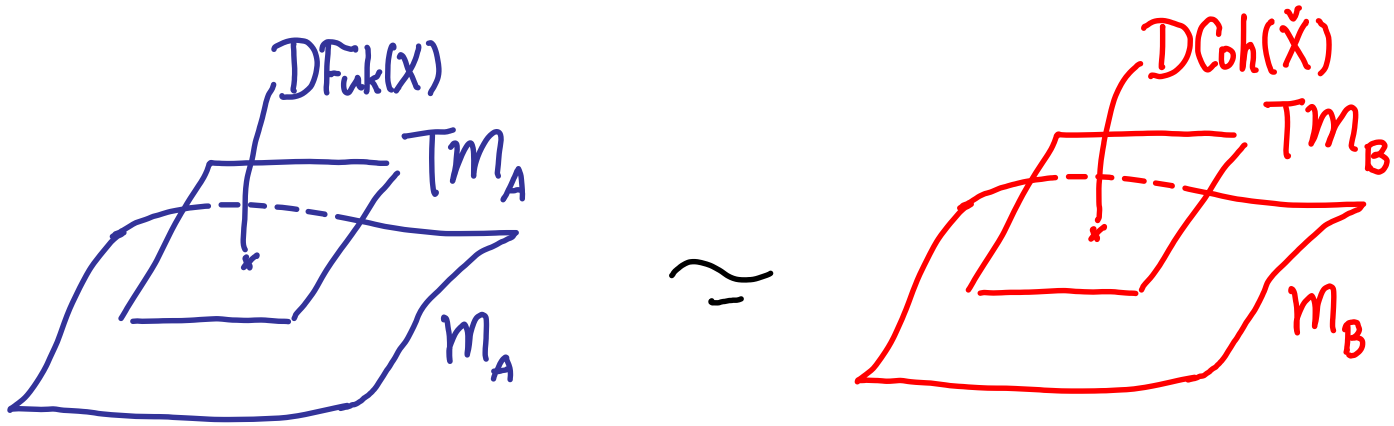
I. Review on mirror construction.

Mirror symmetry

Symplectic geometry $\xleftrightarrow[\text{mirror}]{}$ Complex geometry

X $\qquad\qquad$ $\check{X}$

[Greene-Plesser; Candelas-de la Ossa-Green-Parkas]



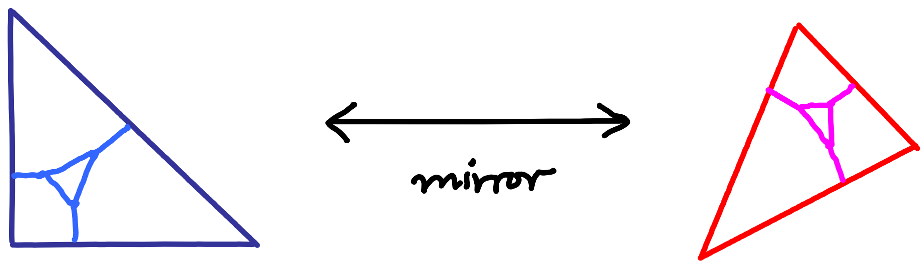
Several levels: Hodge diamonds, Frobenius structures and quantizations,

Homological mirror symmetry.

Why does mirror symmetry occur?

Combinatorial construction of mirror symmetry

Batyrev-Borrisov: construct mirror pairs by dual polytopes.



Berglund-Hübsch: construct mirror pairs by matrix transpose.

$$M \xleftrightarrow{\text{mirror}} M^t$$

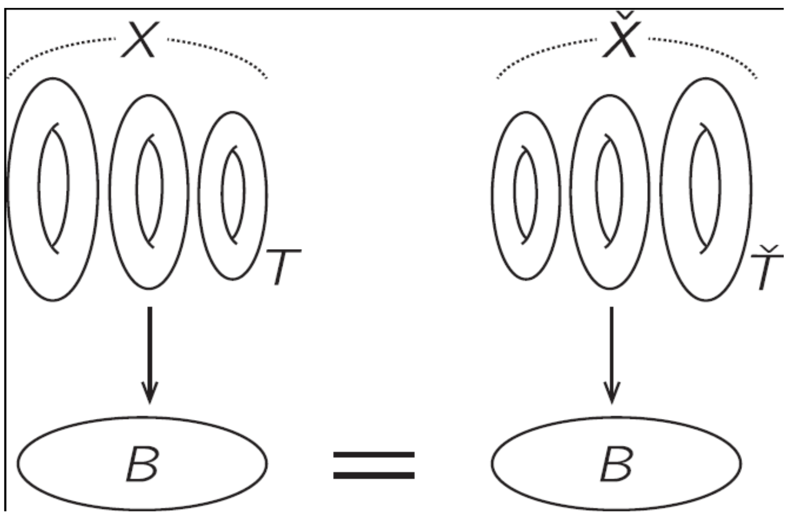
(powers of terms of the defining polynomial)

Why mirror symmetry occurs geometrically is not clear from construction. We want to have a construction such that mirror phenomenons can be directly understood from the construction.

SYZ mirror symmetry program

Strominger-Yau-Zaslow: Mirror symmetry is T-duality.

In particular, mirror can be constructed by taking fiberwise dual of Lagrangian torus fibrations. It gives a concrete geometric understanding of mirror symmetry.



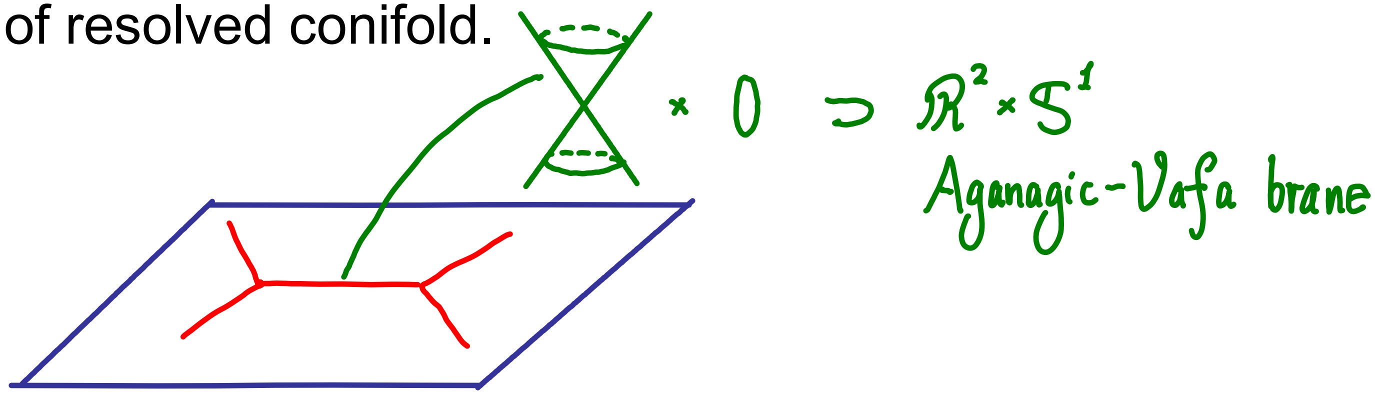
Kontsevich-Soibelman
Gross-Siebert
Leung-Yau-Zaslow
Cho-Oh
Auroux
Abouzaid
Fang-Liu-Treumann-Zaslow
Nadler-Zaslow
Fukaya-Oh-Ohta-Ono
Chan-L.-Leung-Tseng
Abouzaid-Auroux-Katzarkov
Chan-Leung-Ma
......

Construct **Fourier-Mukai transform (Leung-Yau-Zaslow, Tu)** or **family Floer functor (Fukaya, Abouzaid)** to transform Lagrangian submanifolds to coherent sheaves of the mirror.

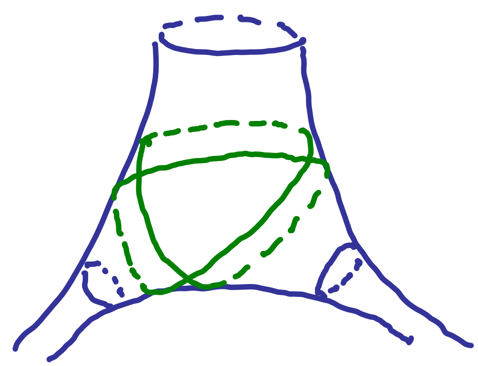
Generalizing SYZ using non-tori

There are cases where Lagrangian torus fibrations are not available or the quantum corrections are too complicated, and other types of Lagrangians are more useful.

Aganagic-Ekholm-Ng-Vafa use non-compact Lagrangian branes arising from knots to construct the mirrors of resolved conifold.



Seidel and Sheridan use Lagrangian immersions in pair of pants (in general dimensions) to prove homological mirror symmetry for genus-two curves and Fermat type Calabi-Yau hypersurfaces. The relations with mirror maps for Abelian varieties were studied by **Zaslow and Aldi-Zaslow**.



Generalizing SYZ using non-tori

Cho-Hong-L.: construct mirrors using a deformation family of immersed Lagrangians. There always exists a functor

$$\mathcal{F} : \mathrm{Fuk}(X) \longrightarrow \mathrm{MF}(W).$$

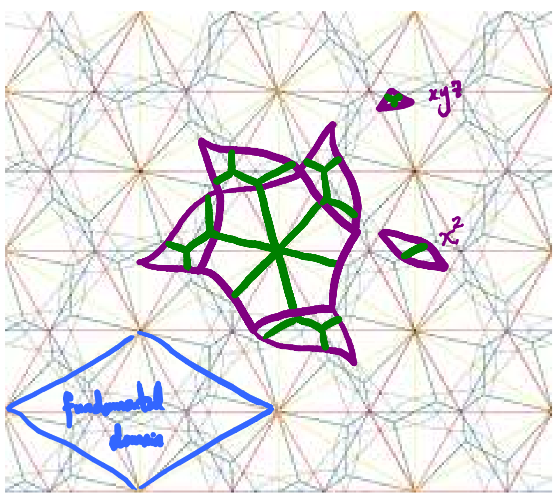
Applied to the Lagrangian immersion constructed by **Seidel** in elliptic and hyperbolic orbifolds, we computed the full mirror superpotential W and showed that the functor derives an equivalence.

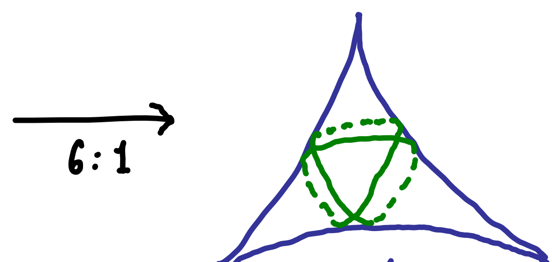
Modularity

L.-Zhou, Bringmann-Rolen-Zwegers: in the elliptic case W is written in terms of (mock) modular forms.

(Modularity is seen only if we include all the higher-order terms.)

e.x.





$\mathbb{P}^1_{2,3,6}$

$$W = q\,xyz + q^6 x^2 + p_1 z^6 + p_2 y^3 + p_3 y^2 z^2 + p_4 y z^4.$$

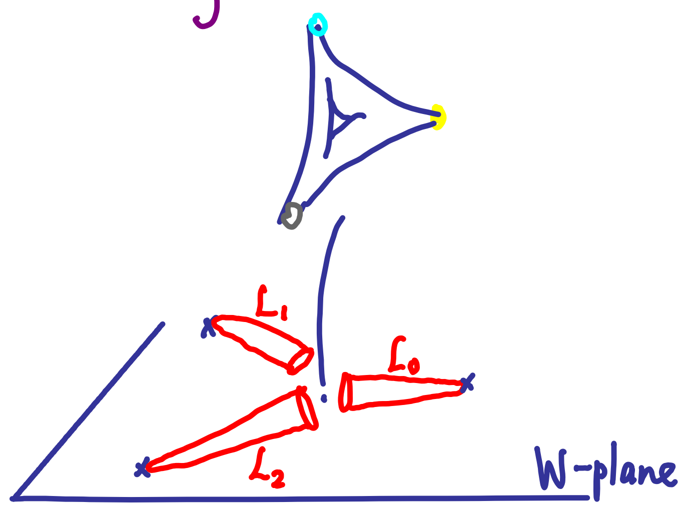
II. Non-commutative mirror construction

Non-commutative mirrors of Auroux-Katzarkov-Orlov

$$(\mathbb{C}^\times)^2,\ W = z + w + \frac{1}{zw} \xleftrightarrow{\text{mirror}} \mathbb{P}^2.$$

$$L_0, L_1, L_2 \in F.S.(W) \simeq D^b Coh(\mathbb{P}^2) \ni \mathcal{O}, \mathcal{O}(1), \mathcal{O}(2)$$

Abouzaid
Fang-Liu-Treumann-Zaslow



Auroux-Katzarkov-Orlov:

non-exact symplectic deformation of $F.S.(W) \simeq D^b(\mathcal{A})$.
(certain compactification of W)

$\mathcal{A}$: Sklyanin algebra [Artin-Schelter]
[Artin-Tate-Van den Bergh]
non-commutative deformation of $\mathbb{P}^2$.

Auroux-Katzarkov-Orlov:

$$\begin{array}{ccc} F.S.(W) & \simeq & D^b(\mathbb{P}^2) \\ \updownarrow & & \updownarrow \\ F.S.(\overline{W}) & \simeq & D^b(\underbrace{\mathcal{A}}_{\text{non-commutative deformation of } \mathbb{P}^2.}). \end{array}$$

Bocklandt:

punctured Riemann surfaces $\xleftrightarrow{\text{mirror}}$ non-commutative LG models constructed from dual dimers.

Non-commutative geometries serve as mirrors in their significant works.

We develop a mirror construction which naturally produces non-commutative geometries. It always comes with a natural injective functor from the Fukaya category to the mirror category of (twisted) bundles.

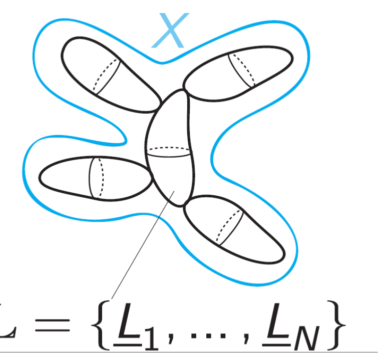
As an application, it can be used to construct explicit deformation quantizations of coordinate rings of algebraic varieties.

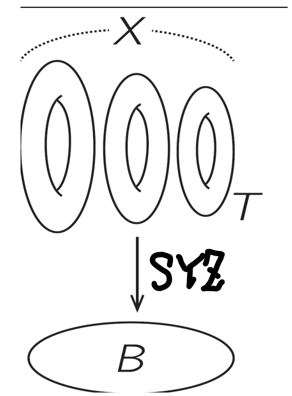
Key points

Construct a mirror using a family of deformations of a finite collection of unobstructed Lagrangian submanifolds.

$$\mathbb{L} \triangleq \{L_1, \dots, L_N\}, \quad L_i \pitchfork L_j \text{ for } i \neq j.$$

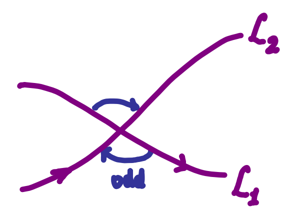
$$L \triangleq \bigcup_{i=1}^{N} L_i.$$





Infinitesimal deformation space:

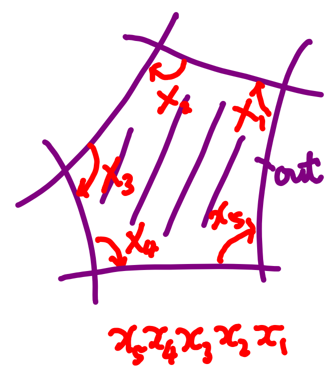
$$V \triangleq \bigoplus_{i \neq j} CF^{odd}(L_i, L_j) \ni b = \sum_a x_a X_a$$



Worldsheet superpotential:

$$W \text{ `}\triangleq\text{'} \sum_\beta n_\beta \, q^\beta \, x^\beta .$$

hol. polygons in β thru a marked point — area — corners of β.



Problem: W is not well-defined since it depends on the position of marked point!

Maurer-Cartan equation for weakly unobstructedness

Fukaya-Oh-Ohta-Ono:

$$m_0^b \triangleq \sum_{k=0}^{\infty} m_k(b,\dots,b) \in CF^{\text{even}}(L,L) \text{ for } b \in V.$$

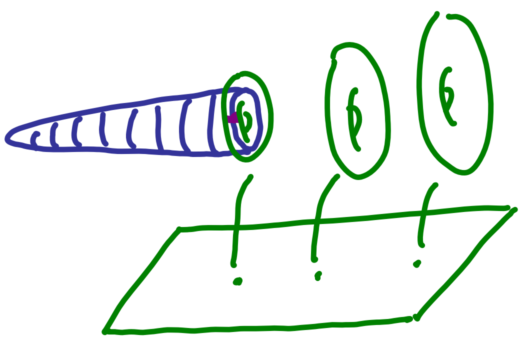
Weakly unobstructedness:

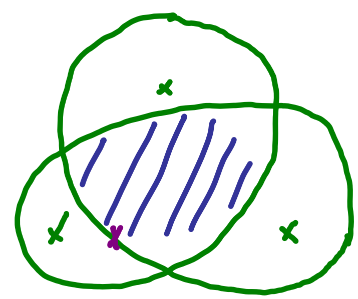
$$m_0^b = W(b) \cdot \mathbb{1}_L .$$

$$\check{X} \triangleq \{ b \in V : m_0^b = W(b) \cdot \mathbb{1}_L \}.$$

Then $(\check{X}, W)$ is a Landau-Ginzburg model.

Fukaya-Oh-Ohta-Ono used this to construct the mirror of compact toric manifolds.
Cho-Hong-L. used this to construct the mirror of elliptic and hyperbolic orbifolds.





Critical problem

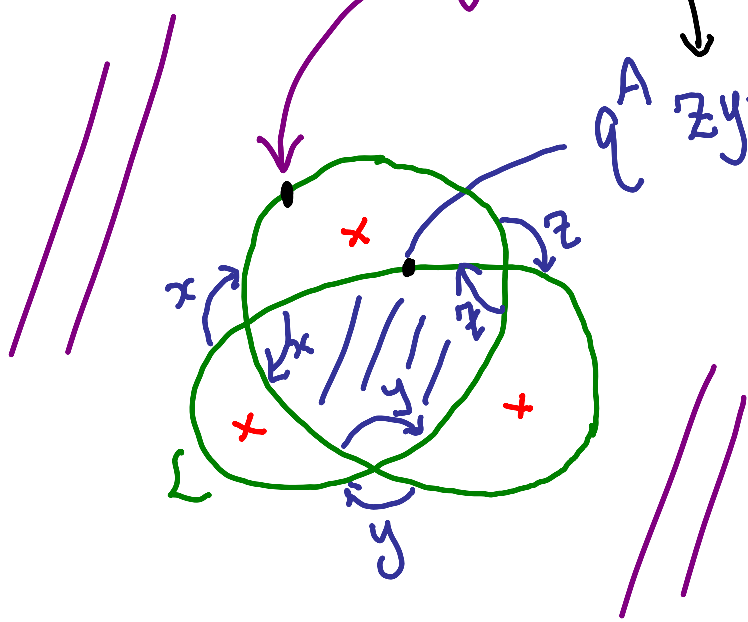
For generic configurations of Lagrangians,
the Maurer-Cartan equation does not have any non-zero solution!

e.g.

$q^{B} xyz$ (outer disc has area B)

not equal when $A \neq B$

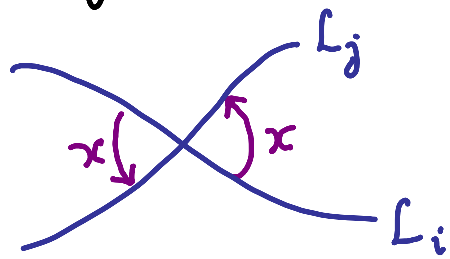
$q^{A} zyx$ (inner disc has area A.)

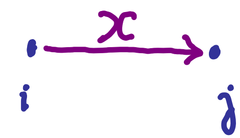


When $A \neq B$, W is not well-defined.

Solution: non-commutative deformations

Take odd morphisms of $L_i \to L_j$ which form a quiver Q.





Take the path algebra of Q in place of V. This records the source and target of each variable x, and the order of variables in each monomial.

Non-commutative deformations: $b \triangleq \sum_a x_a X_a$. ($X_a$ are odd morphisms.)

Maurer-Cartan equation: $m_0^b = W_1(b) \cdot \mathbb{1}_{L_1} + \ldots + W_N(b) \cdot \mathbb{1}_{L_N}$.

$A \triangleq \Lambda Q / R$, R is generated by weakly unobstructed relations.

$W \triangleq W_1 + \ldots + W_N$.

Solution: non-commutative deformations

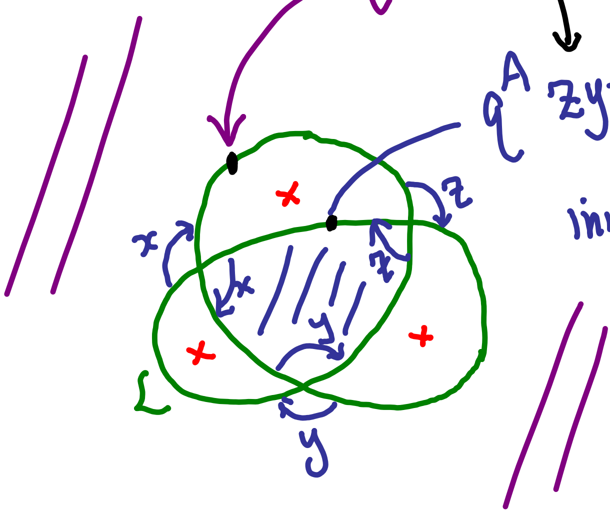
outer disc has area B.

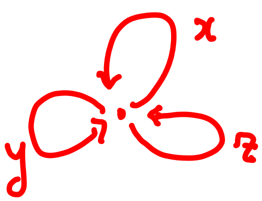
$q^B xyz$

not equal when $A \neq B$

$q^A zyx$

inner disc has area A.



Q: 

. $\Lambda Q = \langle x, y, z \rangle$.

Weakly unobstructed relations: $R = \langle q^A yx \sim q^B xy, \dots \rangle$.

Then $W = q^A zyx = q^B zxy = q^A xzy = q^B xyz$ is well-defined over $A = \Lambda Q / R$.

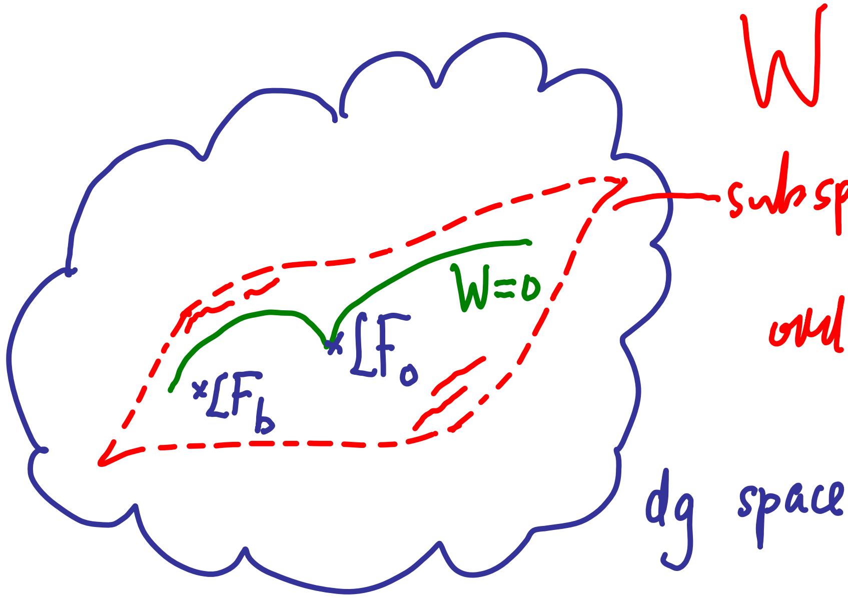
The DG space of Kontsevich and Soibelman

Kontsevich and Soibelman formulated a differential-graded algebra from each A-infinity algebra, which can be considered to be a universal deformation space.

In our case the A-infinity algebra comes from Lagrangian Floer theory.

We only take the odd-degree deformations of the Lagrangian. Then the disc-countings are easier to handle and more computable.

We restrict only to the subspace of weakly unobstructed deformations and work with the formal family of Floer theory over this subspace. As a result, we obtain a (non-commutative) Landau-Ginzburg model rather than a dg algebra.



W defined on subspace of weakly unobstructed odd-degree deformations

Non-commutative Landau-Ginzburg mirror

$\left(\mathcal{A} = \Lambda Q/_{R},\ W\right)$ forms a non-commutative Landau-Ginzburg model.

Theorem: W is always a central element.

$\vdash$: $W \cdot x = x \cdot W$.

$$0 = m^2(e^b) \quad (A_\infty \text{ relations})$$

$$= m(e^b, \underbrace{m_0^b}_{W\cdot\mathbb{1}}, e^b) \quad (\text{weakly unobstructed})$$

$$= m(\underbrace{b}_{\sum x_a X_a}, W\cdot\mathbb{1}) + m(W\cdot\mathbb{1}, b)$$

$$= \sum_a \underbrace{(W\cdot x_a - x_a \cdot W)}_{=0} X_a. \quad (\mathbb{1} \text{ is unit.})$$

Non-commutative v.s. commutative mirror

Non-commutative mirror **(A,W)** fits well for the purpose of mirror symmetry:

1) Symplectic geometry of X is transformed to algebraic geometry of (A,W).

2) Gromov-Witten invariants of X can be extracted from deformations of (A,W).

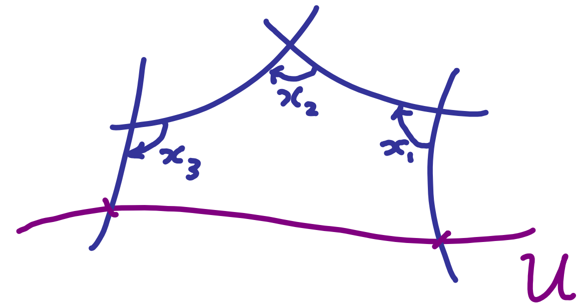
3) (A,W) is more "linear" and easier to work with than an algebraic variety.

4) It is unavoidable for certain Kahler structures of X.

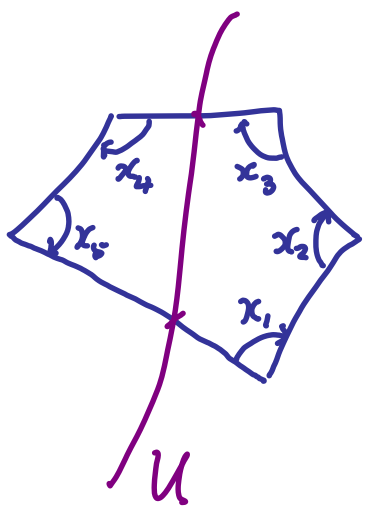
On the other hand, algebraic varieties are more classical and well understood. To go back from non-commutative to commutative geometry, we need to consider moduli space of stable modules in a suitable class. This will be a future research direction.

Mirror functor

$$\exists\ \mathcal{F} : \mathrm{Fuk}(X) \xrightarrow[A^\infty]{} MF(\mathbb{A}, W)$$



$$\mathcal{U} \longmapsto m_1^{(\mathbb{L}, b), \mathcal{U}} : \mathrm{Span}(\mathbb{L}, \mathcal{U}) \circlearrowleft$$



$$\longmapsto \begin{pmatrix} & x_3 x_2 x_1 \\ x_5 x_4 & \end{pmatrix} = M.$$

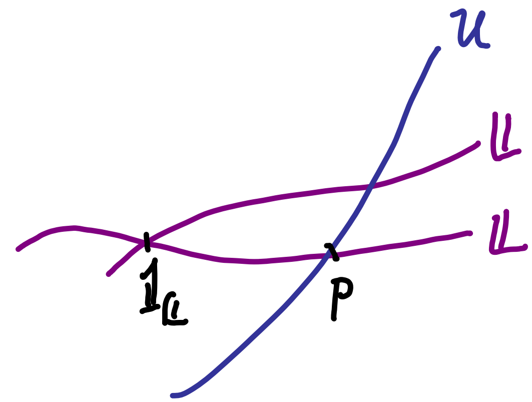
$$M^2 = x_5 x_4 x_3 x_2 x_1.$$

Precisely it uses A-infinity equations and weakly unobstructedness. It is a non-commutative extension of the argument by **Fukaya-Oh-Ohta-Ono**.

Injectivity in morphism level

Theorem: $\mathrm{Hom}(\mathbb{L},\mathcal{U}) \xrightarrow{\ F\ } \mathrm{Hom}(F(\underset{\substack{\cup \\ \mathbb{1}_{\mathbb{L}}}}{\mathbb{L}}),F(\mathcal{U}))$ is injective.

Pf: $\overset{\cup}{\phi(\mathbb{1}_{\mathbb{L}})}\Big|_{b=0} \xleftarrow{\quad\Psi\quad}\!\shortmid\ \overset{\cup}{\phi}$



Ψ is a chain map:

$$
\begin{aligned}
\Psi(m_1(\phi)) &= \Psi(m_1^{b,0}\circ\phi)) - (-1)^{|\phi|}\Psi(\phi\circ m_1^{b,0}) \\
&= \big(m_1^{b,0}(\phi(\mathbf{1}_{\mathbb{L}}))\big)|_{b=0} - (-1)^{|\phi|}\big(\phi(m_1^{b,0}(\mathbf{1}_{\mathbb{L}}))\big)|_{b=0} \\
&= \big(m_1^{0,0}(\phi(\mathbf{1}_{\mathbb{L}})|_{b=0})\big) - (-1)^{|\phi|}\big(\phi(-b)\big)|_{b=0} \\
&= m_1^{0,0}(\Psi(\phi)).
\end{aligned}
$$

$\Psi\circ F = \mathrm{Id}$: $\forall p \in \mathrm{Hom}(\mathbb{L},\mathcal{U})$,

$F(p) \triangleq m_2^b(\cdot\,, p) \in \mathrm{Hom}(F(\mathbb{L}),F(\mathcal{U}))$.

$\Psi(F(p)) = m_2^b(\mathbb{1}_{\mathbb{L}}, p) = p$.

Deformation quantization

$$\mathbb{L} \underset{\text{Lag.}}{\subset} X \longrightarrow (\mathcal{A}, W).$$

$$\mathrm{Fuk}(X) \longrightarrow MF(\mathcal{A}, W) \simeq MF(Y, W).$$

Deform:

$$\mathbb{L}_u \underset{\text{Lag.}}{\subset} X \longrightarrow (\mathcal{A}_u, W_u).$$

Suppose $\mathcal{A}$ is equivalent to the morphism algebra of a certain full exceptional collection of vector bundles over a commutative variety Y (for instance by considering moduli space of quiver representations).

$\mathcal{A}_u$ can give deformation quantizations of Y,

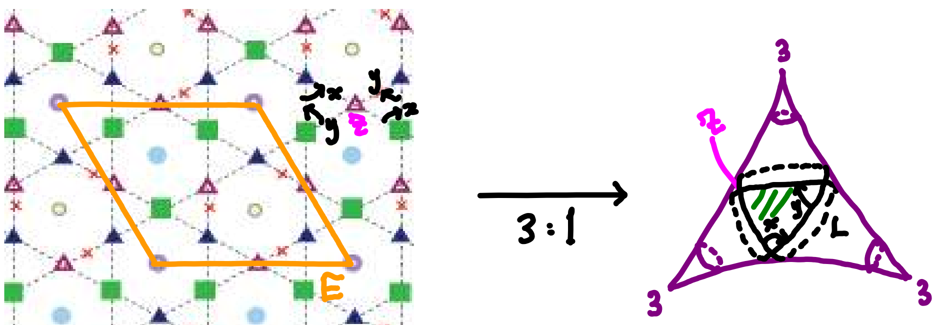
(even though $MF(\mathcal{A}_u, W_u)$ remains the same, since we do not deform ω !)

defined over global moduli of $\mathbb{L}_u$.

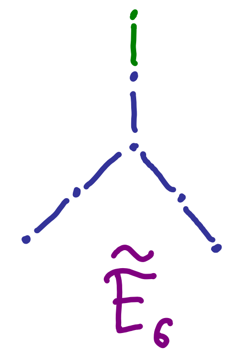
i. Sklyanin algebra and its central element.

The mirror of the elliptic orbifold

$E/\mathbb{Z}_3 = \mathbb{P}^1_{3,3,3}$.



L was constructed by **Seidel**.



$E/\mathbb{Z}_3$ is the elliptic orbifold of type $\tilde{E}_6$.

We fixed a Lagrangian L to construct its mirror.

Important: L was chosen such that it is symmetric about the equator.

Output m_0^b at Z : $yx - xy = 0$.

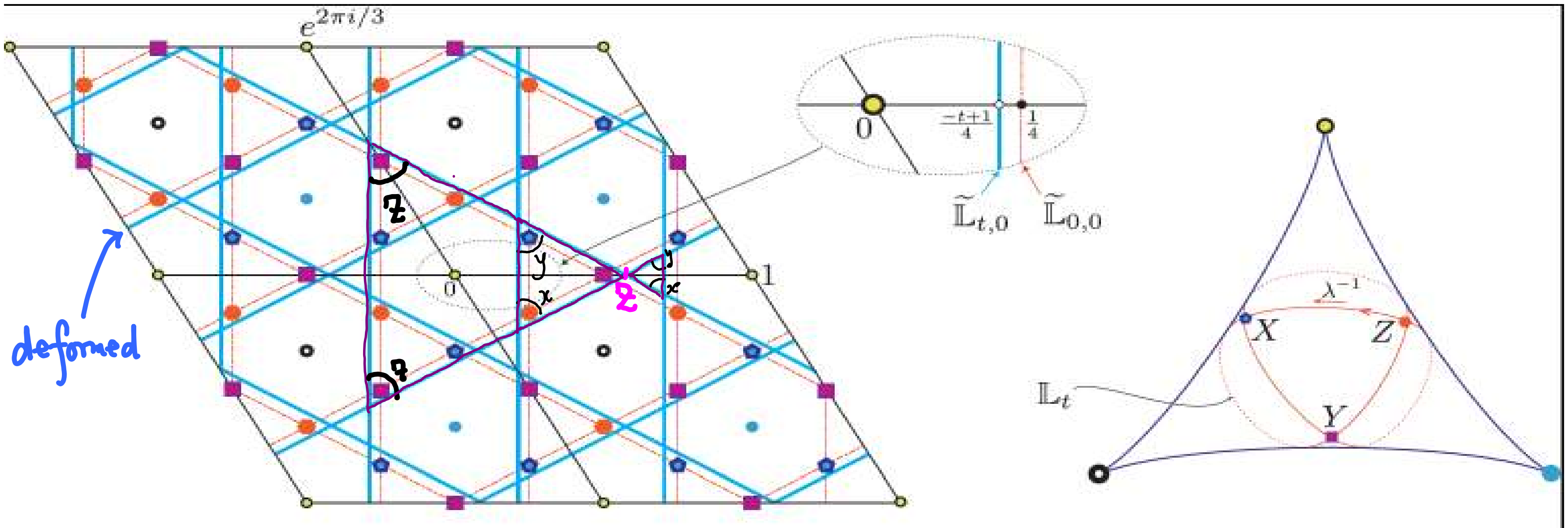
As a result, b = x X + y Y + z Z is weakly unobstructed (need non-trivial spin structure).

Mirror: $(\mathbb{C}^3, W)$, where $W = \varphi(q)(x^3+y^3+z^3) + \psi(q)\, xyz$,

$$\phi(q) = -\eta(q^3)^3,$$
$$\psi(q) = -(\eta(q^{\frac{1}{3}})^3 + 3\eta(q^3)^3).$$

← need all higher-order terms to obtain modularity

Non-commutative deformations



Now deform L such that it is no longer symmetric about the equator.

Output m_0^b at Z: $a\,yx - b\,xy + c\,z^2 \neq 0.$

$(a \neq b\ ,\ c \neq 0.)$

The Maurer-Cartan equation for weakly unobstructedness has no non-zero solution!
We are forced to consider non-commutative deformations.

$$\mathcal{A} \triangleq \mathbb{C}\langle x, y, z\rangle \Big/ \langle a\,yx - b\,xy + c\,z^2,\ a\,zy - b\,yz + c\,x^2,\ a\,xz - b\,zx + c\,y^2 \rangle$$

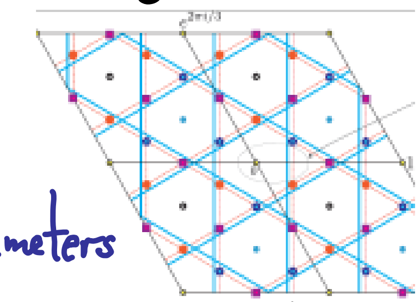
Sklyanin algebra

Central element of Sklyanin algebra

Theorem:
The space of weakly-unobstructed odd-degree deformations is given by

$$\mathcal{A}_w \triangleq \mathbb{C}\langle x,y,z\rangle \Big/ \langle a\,yx - b\,xy + c\,z^2,\ a\,zy - b\,yz + c\,x^2,\ a\,xz - b\,zx + c\,y^2 \rangle$$

where
$$\begin{cases} a(w) = \sum_{k\in\mathbb{Z}} e^{\pi i(3\tau)k^2} e^{2\pi i k(3w)} = \theta[0,0](3w,3\tau) \\ b(w) = \sum_{k\in\mathbb{Z}} e^{\pi i(3\tau)(k+\frac{2}{3})^2} e^{2\pi i(k+\frac{2}{3})(3w)} = \theta\left[\frac{2}{3},0\right](3w,3\tau) \\ c(w) = \sum_{k\in\mathbb{Z}} e^{\pi i(3\tau)(k+\frac{1}{3})^2} e^{2\pi i(k+\frac{1}{3})(3w)} = \theta\left[\frac{1}{3},0\right](3w,3\tau). \end{cases}$$



Kähler parameters

(Im(w) parametrizes the actual deformations,
Re(w) parametrizes the flat U(1) connections.)

$\Rightarrow [a:b:c] : E^\vee \xrightarrow{\sim} \{W=0\} \subset \mathbb{P}^2$.

$\therefore$ Non-commutative deformations are parametrized by points in the elliptic curve. [Artin-Tate-Van den Bergh]

The deformed superpotential equals to

$$W_w = -\frac{\pi i \lambda^{\frac{1+3t}{2}}}{6}\left(a'(w)(xyz+zxy+yzx) + b'(w)(zyx+xzy+yxz) + c'(w)(x^3+y^3+z^3)\right).$$

In particular, W_w is a central element of $\mathcal{A}_w$. (It is difficult to check directly!)

$(\mathcal{A}_w, W_w)$ is a non-commutative mirror of $E/\mathbb{Z}_3$.

Deformation quantization of affine del Pezzo surface

$\mathbb{C}[x,y,z]/(W)$ is the function algebra of $\{W=0\}$. — affine del Pezzo surface

$$W = \varphi(q)(x^3+y^3+z^3) + \psi(q)\, xyz$$

Theorem [**Kontsevich**]: There exists a deformation quantization associated to every Poisson structure of the manifold.

Here $\mathcal{A}_w/(W_w)$ gives deformation quantization of $\{W=0\}$.

$(w \in E^\vee)$

[**Etingov-Ginzburg**: noncoummtative del Pezzo surface]

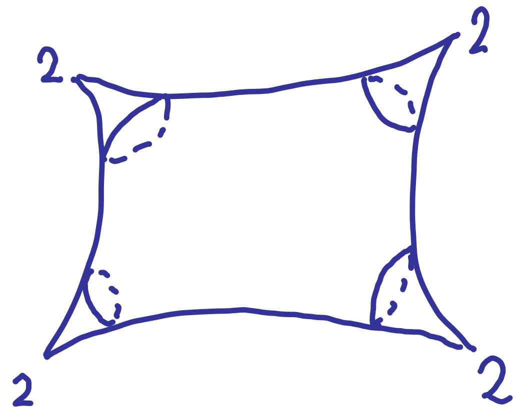
Prop.: $\mathcal{A}_w/(W_w)$ corresponds to the Poisson structure

$$\{f,g\} \triangleq df \wedge dg \wedge dW.$$

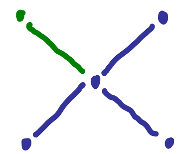
Here we have a global family of non-commutative algebras, written explicitly in terms of theta functions.

ii. Mirror of the pillowcase

The pillowcase $E/\mathbb{Z}_2 \simeq \mathbb{P}^1_{2,2,2,2}$.



$E/\mathbb{Z}_2$ is the elliptic orbifold of type $\widetilde{D}_4$.

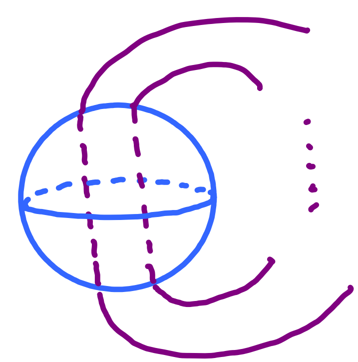


Hedden-Herald-Kirk:
It has close relation with knot and gauge theory:
the pillowcase is the moduli space of traceless flat connections over S^2.

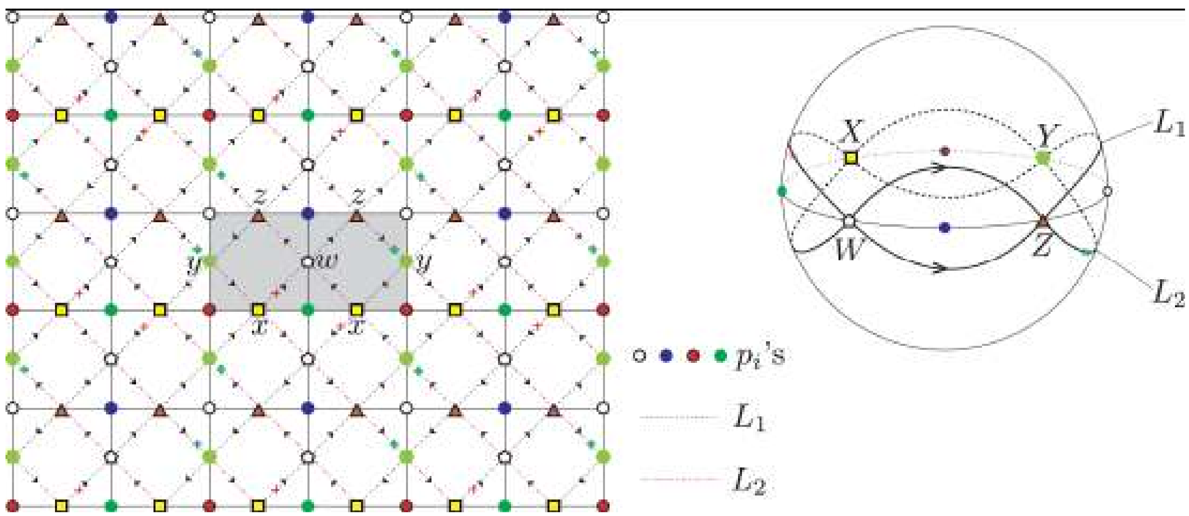
By **Atiyah-Floer conjecture**, instanton Floer homology of a knot is isomorphic to

$$HF(L_1, L_2) \text{ of } E/\mathbb{Z}_2$$

which can be studied through mirror symmetry.

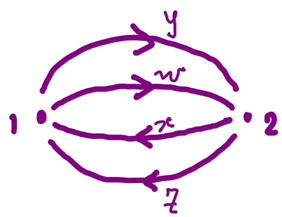


A non-commutative mirror



We fix $\mathbb{L} = \{L_1, L_2\}$, which is symmetric about the equator, to construct its mirror.

It corresponds to the quiver Q:



Output m_0^b at W : $zyx - xyz$.

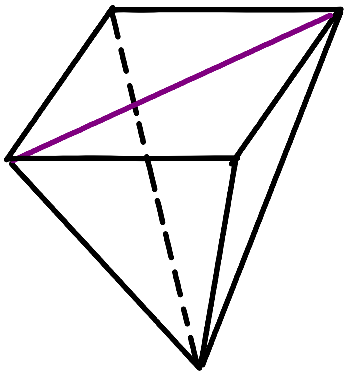
$$A = \Lambda Q \big/ \langle zyx - xyz,\ wzy - yzw,\ xwz - zwx,\ yxw - wxy \rangle .$$

$$W = \phi(q)\left((xy)^2 + \dots\right) + \psi(q)\left(xyzw + wzyx\right)$$

where $\phi = \dfrac{\eta(q)^2\, \eta(q^2)^4}{\eta(q^{1/2})^2}$, $\psi = \dfrac{\eta(q)^{14}}{\eta(q^{1/2})^6\, \eta(q^2)^4}$.

Modularity of the superpotential was studied by a joint work with **Jie Zhou**.

Non-commutative resolution of the conifold



$$Y = \mathcal{O}_{\mathbb{P}^1}(-1)\oplus\mathcal{O}_{\mathbb{P}^1}(-1) = \mathbb{C}^4 \big/\!\!\big/ \langle (x,y,z,w)\sim(\lambda x,\lambda^{-1}y,\lambda z,\lambda^{-1}w)\rangle$$

$$\big\downarrow \text{resolution}$$

conifold $\{X_1X_3 = X_2X_4\}\subset\mathbb{C}^4$

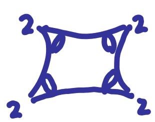
$$\begin{pmatrix} X_1 = yx \\ X_2 = yz \\ X_3 = wz \\ X_4 = wx \end{pmatrix}$$

$\mathcal{O}_Y(1) \overset{y,\,w}{\underset{z,\,x}{\rightleftarrows}} \mathcal{O}_Y$. Then $\underbrace{zyx = xyz\,, \ldots\ldots}_{\text{same relations as in } \mathcal{A}.}$

$\mathcal{A}$ is the non-commutative resolution of the conifold.

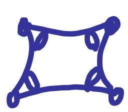
$$D^b(\mathcal{A}) = D^b(Y). \qquad \text{[Bondal-Orlov ; Aspinwall]}$$

In particular



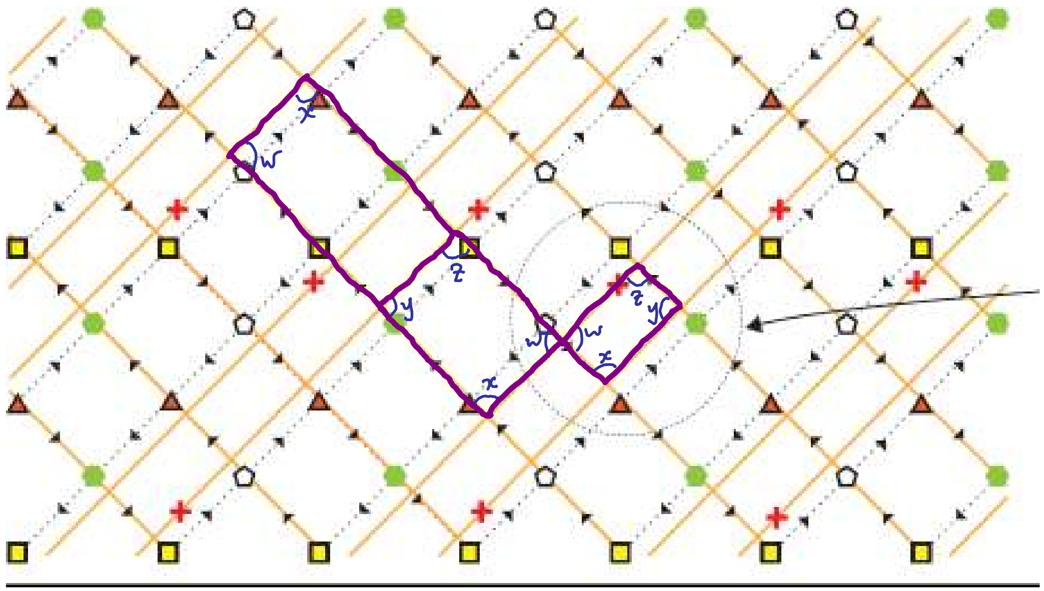
$$E/\mathbb{Z}_2 \xleftrightarrow{\text{mirror}} \left(Y,\ W = \phi(q)\,(X_1^2+X_2^2+X_3^2+X_4^2)+\psi(q)\,X_1X_3\right).$$

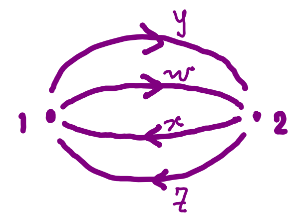
It is compatible with the result of **Abouzaid-Auroux-Efimov-Katzarkov-Orlov** that



$$\mathbb{P}^1 - 4 \text{ points} \xleftrightarrow{\text{mirror}} \left(Y,\ W_0 = X_1X_3\right).$$

Non-commutative deformations



$Q:$ 

Output m_0^b at w:

$$h_w = Axyz + Bzyx + Cxwx + Dzwz.$$

$$\mathcal{A}_u = \Lambda Q / \langle h_x, h_y, h_z, h_w \rangle.$$

$$KA = \frac{\theta[0,0](2u,2\tau)}{\theta\left[\frac{3}{4},0\right](2u,2\tau)}, \quad KB = i\frac{\theta[0,0](2u,2\tau)}{\theta\left[\frac{1}{4},0\right](2u,2\tau)}, \quad KC = \frac{\theta\left[\frac{2}{4},0\right](2u,2\tau)}{\theta\left[\frac{1}{4},0\right](2u,2\tau)}, \quad KD = i\frac{\theta\left[\frac{2}{4},0\right](2u,2\tau)}{\theta\left[\frac{3}{4},0\right](2u,2\tau)}$$

$\mathcal{A}_u$ gives a global family of non-commutative deformations of the resolved conifold.

Ψ central

W_u

$\mathcal{A}_u / \langle W_u \rangle$ gives a global family of non-commutative deformations of the affine del Pezzo surface of type $\tilde{D}_4$: $\{W = 0\} \subset$ resolved conifold.

iii. CY threefolds associated to SL(2) Hitchin system

The Calabi-Yau geometry

[Diaconescu-Donagi-Pantev, Smith]

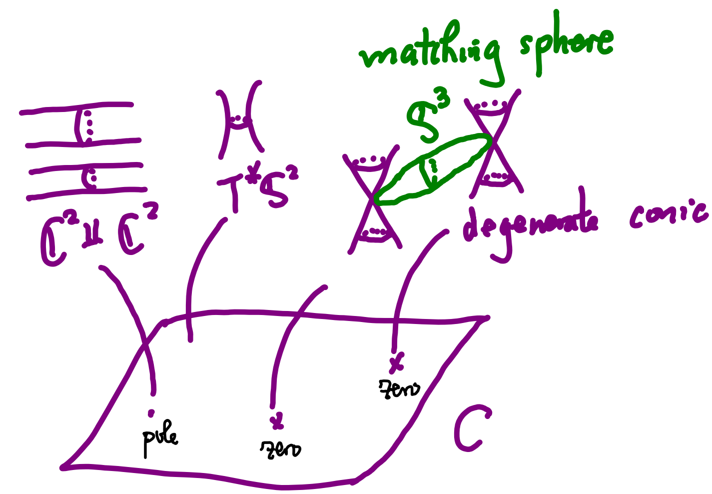
C: a curve with genus $\geqslant$ 1.
P: a set of marked points. $|P| \geqslant 3$.

ϕ: quadratic differential on C with double poles at P. (It comes from the tensor product of the two eigen-one-forms of a SL(2) Higgs field on C.)

The Calabi-Yau threefold is essentially defined by $X = \{uv = \eta^2 + \phi(x)\} \xrightarrow[\text{conic fibration}]{} C.$

Special Lagrangian matching spheres are the BPS states studied by **Gaiotto-Moore-Neitzke**.
The countings of BPS states undergo wall-crossing as the quadratic differential varies, which obey **Kontsevich-Soibelman's formula**.

Bridgeland-Smith deduced the wall-crossing by embedding derived category of certain **Ginzburg algebra** to the Fukaya category, and identifying quadratic differentials with stability conditions on the derived category. The stability moduli has cluster structure studied by **Goncharov**.



Our general framework can be applied to this situation as well. It produces a functor from the Fukaya category to the derived category of the Ginzburg algebra, which turns out to be a right inverse of Smith's embedding.

WKB collection of Lagrangian spheres

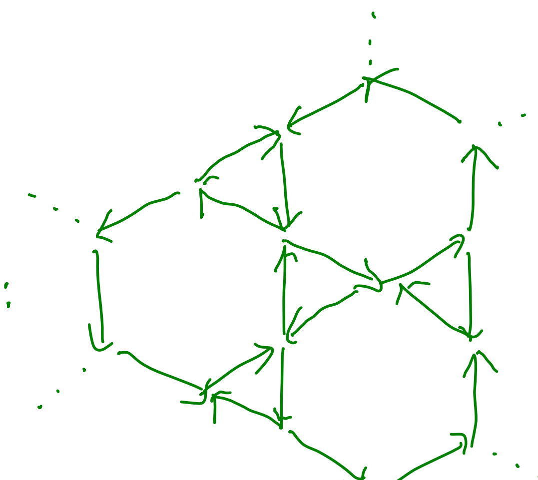
The quadratic differential ϕ induces a triangulation of C, whose vertices is the prescribed set P.

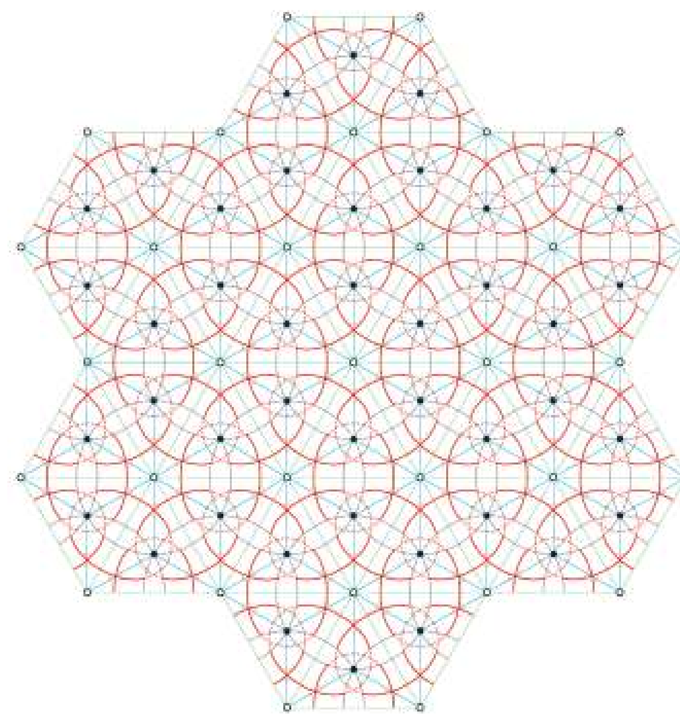
From the triangulation we can construct a basis $\mathbb{L}$ of Lagrangian spheres, which is called the WKB collection.

The Floer endomorphisms of $\mathbb{L}$ in degree one produce a quiver Q, which can be identified as a dimer (embedded bipartite graph) in C.

The basis of Lagrangian spheres can be constructed from zig-zag Lagrangian circles made from the dual dimer in the spectral curve.

Q :





Vertices correspond to WKB Lagrangians;
Arrows correspond to degree-one morphisms.

Unobstructed relations

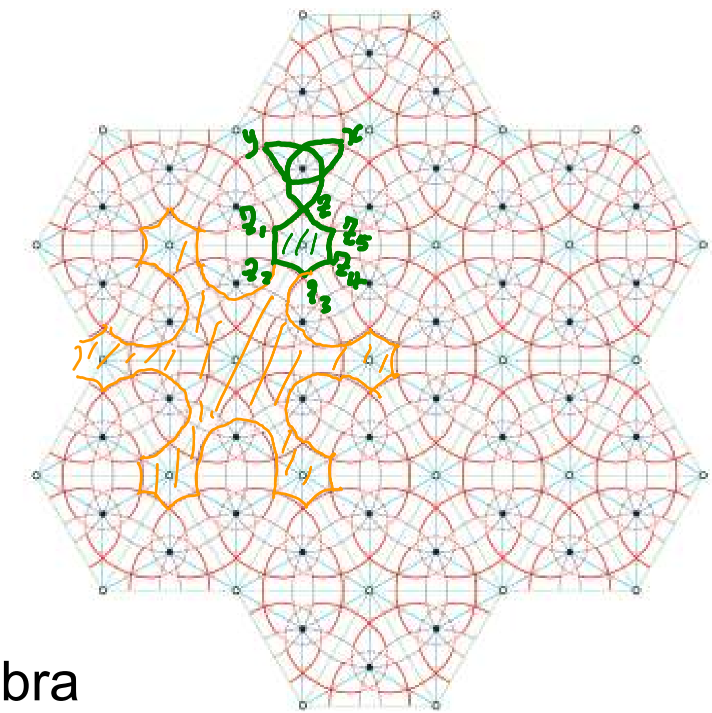
In this case, due to degree reason, the (worldsheet) superpotential W = 0.

(m_0^b has degree 2, but $\mathbb{1}_L$ has degree 0.)

Coefficient of m_0^b at z : (technical sign issue involved)

$$xy + c\, z_5 z_4 z_3 z_2 z_1 + \text{higher-order terms}$$

$$= \partial_z \left(zxy + c\, z z_5 z_4 z_3 z_2 z_1 + \text{higher-order terms} \right).$$



$$\mathcal{A} = \Lambda Q \big/ \langle \partial \Phi \rangle$$ Jacobian of Ginzburg algebra

where $\Phi \underset{\text{[Smith]}}{=} \underbrace{\sum (\text{triangle terms}) + \sum (\text{polygon terms})}_{\Phi_0} + \ldots\ldots$

spacetime superpotential

(Smith used Φ_0 , whose corresponding category is quasi-equivalent to that of Φ .)

The mirror functor

Theorem: The mirror functor $DF : D^{\pi}Fuk(X) \longrightarrow D^b(A)$

restricts to be an equivalence from the subcategory generated by matching Lagrangian spheres to $D^b(A)$.

Steps of proof:
(1) Compute the images of the WKB Lagrangians under the functor. We show that in the cohomological level, the WKB Lagrangians are transformed to simple modules over the corresponding vertices of the quiver.

(2) Compare the dimensions of morphism spaces. From the result of **Keller-Yang**, the endomorphism space of a simple module is two-dimensional, which matches with the Floer cohomology of a Lagrangian sphere. Moreover the morphism space of two adjacent vertices of an arrow is one dimensional, which matches with the intersection number of the corresponding Lagrangians.

(3) Since the functor is always injective on morphism spaces, we conclude that it is isomorphism on morphism spaces.

(4) By **Keller-Yang**, the derived category is generated by the simple modules. Result follows.